# Healthcare Services Optimization by Fusing Predictive and Prescriptive Analytics for Multi-Site Modelling

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***ABSTRACT:*** *Healthcare models must deliver personalized, actionable insights in real time for both patients and doctors to aid their treatment choices. A patient-centered approach is essential for merging EHR data, patient information, prescriptions, monitoring, and clinical research data. The significance of combining prescriptive and predictive analytics in the healthcare sector is emphasized in this research. The proposed model will extract meaningful insights to facilitate decision-making and enable real-time medical predictions and prescriptions through advanced process analysis. We examined the potential of "predictive and prescriptive analytical solutions through Multi-Site Modeling to enhance healthcare services". Clinical and demographic patient data are collected and utilized to generate predictive outcomes, such as estimated length of stay and initiation of medication for treatment, among others. Two-stage stochastic and Deterministic models were created on the prescriptive end to optimally organize the allocation of beds and ward personnel while reducing costs. We used Classification and Regression Trees (C A R T) analysis to investigate this link The results indicate that fusing predictive and prescriptive analytics significantly enhances decision-making across multiple healthcare sites, leading to optimized resource allocation, improved patient outcomes, and greater operational efficiency.*

***Keywords****- Healthcare, predictive, prescriptive, clinical records, Deterministic model, two-stage stochastic model, CART.*

## INTRODUCTION

With the evolution of time and technological advancements, it is essential to implement systematic changes in health systems to enhance patient care by increasing its quality, efficiency, and effectiveness. Chronic illnesses such as heart disease, diabetes, stroke, and cancer rank among the most prevalent, costly, and preventable health issues. However, due to inadequate healthcare systems, doctors struggle to adequately address their patient's needs. The aim of value-based health care is to guarantee that everyone has access to the necessary health services for their well-being, which aligns with a greater focus on patient-centered care. There needs to be an improvement in the quality and coordination of healthcare to ensure that patient outcomes are consistent with current professional standards. The expenses associated with treating health issues should be lowered so that all patients can receive personalized treatment efficiently and at a reduced cost. Howsoever, It is quite challenging to effectively utilize the vast amounts of unstructured data from various sources to make timely decisions for individual patients by healthcare providers. This can hinder the delivery of personalized care to patients. Therefore, developing a new approach or framework for patient care that emphasizes their well-being while minimizing healthcare costs is crucial.

***Predictive Modeling Techniques:*** To predict unfavorable health outcomes in patients, machine learning models like decision trees, classification trees, and regression trees are used.

***Prescriptive Analytics Techniques****:* To recommend optimal healthcare interventions, prescriptive analytics leverages optimization algorithms that evaluate various treatment scenarios. These techniques are used to personalize medical decisions—such as drug regimens—with the goal of improving clinical effectiveness and minimizing adverse effects, thereby supporting data-driven, patient-centered care.

***Data Collection Methods:*** Comprehensive patient information, such as demographics, medical history, and current prescriptions, is provided by electronic health records, or EHRs. Real-time health indicators are provided via wearable technology and remote monitoring systems, which support ongoing patient evaluation.

***Evaluation Metrics:*** The model's effectiveness is evaluated using performance metrics like accuracy and precision. The efficacy of recommended therapies is assessed by looking at clinical outcomes, such as lower hospitalization rates and better patient quality of life.

***Tools and Technologies:*** Python and other programming languages are used to create the component of this hypothetical analytical model to examine the case outcomes.

The work's conclusions, which highlight how important it is to consider a "fusing of predictive and prescriptive analytics" in order to make informed decisions, will be very helpful to healthcare executives. In the end, this strategy may result in improved health outcomes and more efficient utilization of resources.

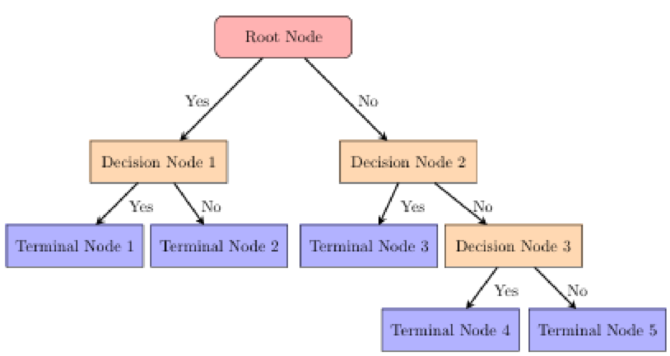
## OBJECTIVE

- ***Lower readmission rates***: There ought to be more efficient and patient-focused strategies for lowering readmission rates. With the help of analytical model advice, doctors may determine which patients are at risk of readmission and take steps to reduce that risk.
- ***Minimize length of stay (LOS):*** Analytical tools help decrease LOS and enhance outcomes such as patient satisfaction by identifying individuals at risk for extended hospital stays and promoting adherence to best practices.
- ***Chronic Disease Prediction***: Analytical tools employ machine learning to detect patients with undiagnosed or incorrectly diagnosed chronic conditions and predict the probability of them developing chronic diseases in the future and provide tailored preventative treatments.

## RELATED WORK

This section seeks to shed light on the body of extant operational research (OR) literature,
with a particular emphasis on hierarchical CART procedures as well as stochastic and deterministic model -ling approaches.

***Applying CART Analysis to Patients:*** Prediction models are constructed from data using a machine learning method known as CART. A decision tree with a hierarchical structure is produced by the algorithm. Using binary recursive partitioning, the decision tree divides each node into two distinct groups based on responses to a set of questions that determine the data categories.




**Example of a decision tree with one root node, three decision nodes and five terminal nodes.**

***Use of CART in Hospitals:* “Byeon (2015)”** used CART models to forecast endocrine problems. By comparing older and younger patients, “**Watanabe et al. (2018)”** used CART to identify important risk factors for rotator cuff injuries. They discovered that the most important influence was age.

***Use of CART in Hospitals and Community Care***: CART models have been applied beyond hospitals to wider healthcare settings.

- **“Kuo et al. (2019)”** built a tool to predict social frailty in older adults using 15 factors (e.g., age, BMI, income, marital status). Random forest and C5.0 models achieved high accuracy (0.970).
- **“Passmore et al. (1993)”** used patient traits to predict unplanned hospital admissions. The Sickness Impact Profile (SIP) score was the strongest predictor. The number of medications also played a role.

These studies show CART is effective for analyzing and providing hospital services to patients. This research will expand on that by studying how different hospital sites affect patient length of stay (LOS), and by including additional data types like radiology.

***Multi-site Deterministic and Stochastic Analytical Models***

Deterministic models are commonly used in healthcare because they are easier to apply, but many healthcare systems behave unpredictably and are better represented by stochastic models **“Mandelbaum et al., (2018)”**.

***Deterministic Models:*** Deterministic models are used to simplify healthcare planning by assuming fixed outcomes.

- **Hare et al. (2009)** created a Markov model for planning home and community care services across five age groups (three covering elderly patients). They accounted for changes in age and health trends to forecast service needs over time.

***Stochastic Models:*** Stochastic models handle uncertainty, making them more suitable for real-world healthcare settings.

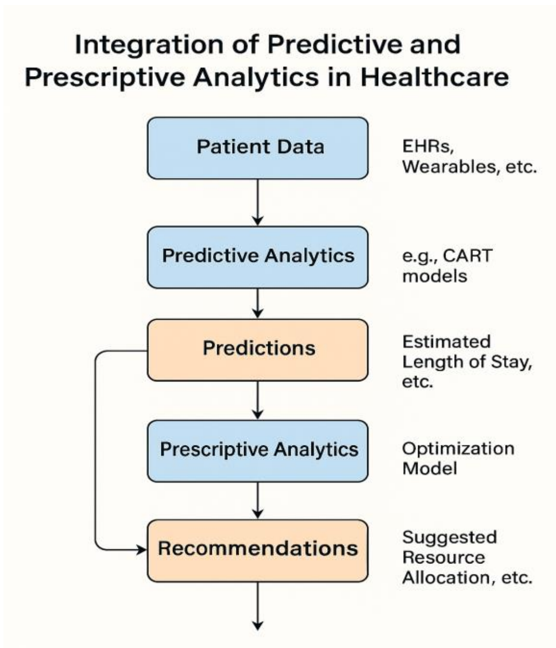
- **Abdelaziz & Masmoudi (2012)** distributed hospital beds and staff among 157 public hospitals using a multi-objective stochastic software. To handle fluctuating demand, they divided specialists into primary, secondary, and tertiary levels.
- **Guo et al. (2021)** optimized surgery schedules using advanced decomposition methods to handle unpredictable factors like surgery duration and cancellations.
- **Thompson et al. (2009)** used Markov decision processes to allocate patients during demand surges, aiming to reduce transfer costs and improve short-term planning.

***Combining Deterministic and Stochastic Models***

**Mestre et al. (2015)** applied both deterministic and stochastic location-allocation models to design hospital networks. Their goal was to improve access to healthcare services while keeping costs low. They found that including both allocation and location decisions in the 1$^{st}$ stage of planning made the model more adaptable. This allowed the second stage to better manage unmet demand and additional capacity needs.

### Main Contributions and Literature Summary

Our literature review highlights a gap in connecting predictive analytics with prescriptive optimization for healthcare resource planning under uncertainty. Most existing work relies on deterministic models, which overlook the variability typical in healthcare systems. Our study addresses this by integrating data-driven predictive modeling (using CART) with two-stage stochastic optimization. As a result, the resource allocation model becomes more resilient and adaptable. Our method makes a significant advance by directly connecting prediction and optimization, in contrast to previous work that handles them independently.




## METHODS USED

Because CART provides a visual representation, it was selected above alternative prediction approaches. Healthcare professionals may now understand and trust the model's output, and there is a chance that they will work together to develop clinically and statistically significant groups. In this work, a mixed integer programming approach is taken into consideration due to the intricacy of our problem.

### *CLASSIFICATION AND REGRESSION TREES*

Classification and regression trees are a data mining technique which is used to forecasts an outcome using variables. The algorithm predicts the value for continuous dependent variables, determines the class for categorical dependent variables, and predicts the value for regression trees. A decision tree gives the parameters that helps to finalize group for visual representation. The decision tree algorithm poses a number of queries in order to determine the categories into which the data is divided. In CART models, each node is split into two groups using binary recursive partitioning. Both classification and regression trees use data that contains categorical variables. Because of the nature of the procedure, the variables need to be preprocessed before they can be converted into numerical data. Since the categorical data does not have an ordinal relationship, it must be transformed to integer encoding using "**one-hot encoding**". Each numeric literal is represented by a separate binary variable, replacing the original categorical parameter. Both the numerical variables and the newly one-hot encoded variables can then be utilized with the CART method.

*1.* **General Formulation -** To identify the optimal split in the algo, CART approaches employ 2 metrics. Regression trees use the MSE (mean squared error) as a criterion, while classification trees use the Gini Index. MSE, which is calculated as follows, informs the user of the degree to which their prediction deviates from the original objective:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

- **n** → The overall count of data points in the dataset.
- **$Y_i$** → The true value of the dependent variable corresponding to the *i-th* data point.
- **$\hat{Y}_i$** → The estimated or predicted value of the dependent variable for the *i-th* data point.
- **$(Y_i - \hat{Y}_i)^2$** → The square of the deviation between the actual and predicted values for the *i-th* observation.

The Gini Index determines the optimal splitting choice for classification trees. The Gini Index generates a number between 0 and 1, where a lower number indicates greater homogeneity in the sample. The Gini Index is calculated in this manner:

$$\text{Gini Index} = 1 - \sum_{i=1}^{n} p_i^2$$

- **n** → The number of distinct categories or classes.
- **pi** → The likelihood or fraction representing an item's membership in class *i*.
- **$pi^2$** → Squared probability of class *i*.
- $\sum^n$ **$pi^2$** → The probability of class *i* raised to the power of two.

**Gini Index** → A value between 0 and 1 that indicates impurity (0 means perfectly pure, 1 means maximum impurity).

*2.* **Feature Specifications-** Several parameters of the CART algorithm can be changed to enhance the model. Until a stopping condition—such as a minimum impurity drop, or a maximum tree depth, or a minimum samples per leaf node is satisfied, and splitting process keeps going recursively. Based on its capacity to lower the prediction error or distinguish between the target variable's classes, a feature parameter is chosen at each internal node to segregate the data.

*3.* **Evaluation Metrics-** To assess each of the numerous CART models' performance and LOS prediction accuracy, a series of evaluation metrics can be applied. Regression trees are assessed using the R2 value. The R2 value is determined as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)}{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)}$$

The parameter representation in above formula:

- **n** → The total number of data entries or observations.
- **Yi** → The observed (actual) value for the *i-th* data point.
- $\hat{Y}_i$ → The predicted value for the *i-th* data point.
- $\bar{Y}_i$ → The average (mean) of all observed values.
- $\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$ → This indicates the sum of squared prediction errors.
- $\sum_{i=1}^{n} (Y_i - \bar{Y})^2$ → This indicates the overall variation in the actual data.
- $R^2$ → The coefficient of determination, which measures the fraction of total variance explained by the model.

An $R^2$ value:

- Close to **1** → Strong predictive power.
- Close to **0** → Weak predictive power.

### Deterministic and Two-Stage Stochastic Programming

Determining nursing staff and total number of beds needed for each case inside each hospital is the model's goal. Each resource was chosen because of their significant influence on the provision of healthcare services. Patient capacity is directly impacted by beds, and nursing personnel play a critical role in guaranteeing both operational effectiveness and high-quality patient care. The distribution of resources is crucially linked since the count of beds required proportional to the staffing requirements and vice versa.

**General Formulation:** Consider a Two-Stage Stochastic Problem, where the decision-makers choose a decision xxx from the global choices space XXX with the goal of minimizing expected costs.

$$\min_{x \in X} E_{\boldsymbol{\xi}} z(x, \boldsymbol{\xi}) = \min_{x \in X} \{f_1(x) + E_{\boldsymbol{\xi}} [h_2(x, \boldsymbol{\xi})]\}$$

- $\boldsymbol{x} \in \boldsymbol{X}$ → First-stage decision variables, where $X \subset R^n$ represents the feasible region. These choices are made prior to knowing the realization of the uncertainty $\xi$.
- $\boldsymbol{f1(x)}$ → The first-stage objective function, which is deterministic and depends solely on the decision variable $x$.
- $\boldsymbol{\xi} \in \boldsymbol{\Omega}$ → A random vector denoting uncertain parameters, defined over a probability space $(\Omega, A, p)$, where:
  - $\Omega \subset R^n$ is the set of all possible outcomes (sample space),
  - $\mathcal{A}$ is a $\sigma$- algebra of events,
  - $p$ is the associated probability measure.
- $\boldsymbol{E}_{\xi}[\cdot]$ → The expectation operator, taken with respect to the probability distribution of the random variable $\xi$.
- $h_2(\boldsymbol{x}, \boldsymbol{\xi})$ → The second-stage (or recourse) function, representing the cost incurred after uncertainty $\xi$, is observed, given the first-stage decision $x$.
- $\boldsymbol{z(x, \xi)}$ → The complete cost function that captures both first- and second-stage costs for a particular realization of $\xi$.

In the next equation we are in second stage of stochastic problem:

$$h_2(x, \xi) = \min_{y \in Y(x,\xi)} f_2(y : x, \xi)$$

- $\boldsymbol{x}$ → First-stage decision variables (chosen before the uncertainty is revealed).
- ξ → Random vector representing uncertainty (e.g., demand, cost, etc.).
- $\boldsymbol{y} \in \boldsymbol{Y}(\boldsymbol{x}, \boldsymbol{\xi})$→ Second-stage (recourse) decisions made *after* observing $\boldsymbol{\xi}$, constrained to the feasible set $\boldsymbol{Y}(\boldsymbol{x}, \boldsymbol{\xi}) \subset \boldsymbol{R^n}$.
- $\boldsymbol{f_2}(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{\xi})$ → Cost function associated with the second-stage decision y, given the first-stage decision $\boldsymbol{x}$ and realization of uncertainty $\boldsymbol{\xi}$.
- $\boldsymbol{h_2}(\boldsymbol{x}, \boldsymbol{\xi})$ → **Recourse function** — represents the optimal cost of adapting to the uncertainty $\boldsymbol{\xi}$, given the earlier decision $x$.

$$RP = E_{\boldsymbol{\xi}} z(x^*, \boldsymbol{\xi})$$

The following equation can be interpreted as the decision-makers substituting the random variables with their expected values, thereby solving a deterministic model. This approach is also referred to as the expected value problem.

$$EV = \min_{x \in X} z(x, \bar{\xi})$$

- $\boldsymbol{z}(\boldsymbol{x}, \boldsymbol{\xi^-})$ → The objective value evaluated using the **expected value** $\bar{\xi} = E[\xi]$ of the random vector $\xi$, rather than accounting for its full distribution.
- $\bar{\boldsymbol{\xi}}$ → The **mean** or **expected value** of the uncertainty $\xi$.
- $\boldsymbol{x} \in \boldsymbol{X}$ → First-stage decision variables, chosen from the feasible set $X \subset R^n$.
- **EV (Expected Value solution)** → Represents the optimal solution obtained when **uncertainty is replaced by its average value**, i.e., a **deterministic simplification** of the full stochastic problem.

***Sets*** or unique collection utilized in the "deterministic and two-stage stochastic analytical models" are shown in the table1. The model variables and parameters are determined by the collection sets. The Sets are defined as follow:

Each specialty should be represented in not less than one hospital ($S \subseteq H$). Likewise, each hospital should be assigned to one of the regions ($H \subseteq R$). Hence, $|R| \geq |H|$.

| Set | Range | Definition |
|---|---|---|
| $\mathcal{B}$ | b = 1,..., B | Set of nursing bands |
| $\mathcal{S}$ | s = 1,..., S | Set of specialties |
| $\mathcal{H}$ | h = 1,..., H | Set of hospitals |
| $\mathcal{R}$ | r = 1,..., R | Set of regions |
| $\mathcal{K}$ | k = 1,..., K | Set of scenarios |

**Table 1** **The sets used within the two-stage stochastic model where (B,S,H,R,K) represent the maximum number of nursing bands, specialties, hospitals, regions and scenarios, respectively.**

***The Parameters Table 2*** outlines the variables employed in both the deterministic and two-stage stochastic models**.**

| Parameter | Definition |
|---|---|
| $c_{s,h}^{\text{bed, 1st}}$ | Cost of the first stage beds for specialty $s \in \mathcal{S}$, in hospital $h \in \mathcal{H}$ |
| $c_{s,h}^{\text{bed, 2nd}}$ | Cost of the second stage bed per day for specialty $s \in \mathcal{S}$, in hospital $h \in \mathcal{H}$ |
| $c_{b}^{\text{staff, 1st}}$ | Cost of the first stage staff of band $b \in \mathcal{B}$ |
| $c_{b}^{staff,\text{ 2nd}}$ | Cost of the second stage staff of band $b \in \mathcal{B}$ |
| $P_k$ | Probability of scenario $k \in \mathcal{K}$ |
| $D_{s,r,k}$ | Demand for each specialty $s \in \mathcal{S}$, arriving from region $r \in \mathcal{R}$, for scenario $k \in \mathcal{K}$ |
| $R_{s,b}$ | Ratio of nursing staff of band $b \in \mathcal{B}$ to patient for each specialty $s \in \mathcal{S}$ |
| $K_{s,h}$ | Number of beds available to open in each specialty $s \in \mathcal{S}$, in hospital $h \in \mathcal{H}$ |
| $UB_h^{max,\ bed.\ 1st}$ | Upper bound of the number of beds that can be deployed in hospital $h \in \mathcal{H}$ in the 1st stage |
| $UB_h^{max,\ bed.\ 2nd}$ | Upper bound of the number of beds that can be deployed in hospital $h \in \mathcal{H}$ in the 2nd stage |
| $UB_b^{max,\ staff.\ 1st}$ | Upper bound of the number of staff that can be deployed in the 1st stage |
| $UB_b^{max,\ staff.\ 2nd}$ | Upper bound of the number of staff that can be deployed in the 2nd stage |

**Table 2** **The parameters used within the two-stage stochastic model where (b,s,h,r,k) represent the maximum number of nursing bands, specialties, hospitals, regions and scenarios, respectively.**

**Decision Variables:** The decision variables presented in table 3 tells the required number of "**beds and nursing staff for each specialty within each hospital**".

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| Decision Variable | Definition |
|---|---|
| $x_{s,h}^{\text{bed}} \in N$ | Number of beds planned in the 1st stage for specialty $s \in \mathcal{S}$ in hospital $h \in \mathcal{H}$ |
| $x_{s,b,h}^{staff} \in N$ | Number of staff planned in the 1st stage for specialty $s \in \mathcal{S}$ ,of band $b \in \mathcal{B}$, in hospital $h \in \mathcal{H}$, |
| $u_{s,r,h,k}^{bed} \in N$ | Number of beds needed in the 2nd stage for specialty $s \in \mathcal{S}$, for patients from region $r \in \mathcal{R}$, in hospital $h \in \mathcal{H}$, for scenario $k \in \mathcal{K}$ |
| $u_{s,b,h,k}^{staff} \in N$ | Number of staff needed in the 2nd stage for specialty $s \in \mathcal{S}$, of band $b \in \mathcal{B}$, in hospital $h \in \mathcal{H}$, scenario $k \in \mathcal{K}$ |

**Table 3 The decision variables used within the two-stage stochastic model where $(b, s, h, r, k)$ represent the value number of nursing bands, specialties, hospitals, regions and scenarios, respectively.**

**Model:** Based on the specified sets, parameters, and decision variables, the deterministic model is formulated as follows:

$$\min \sum_{h\in\mathcal{H}} \sum_{s\in\mathcal{S}} (c_{s,h}^{\text{bed}} x_{s,h}^{\text{bed}} + \sum_{b\in\mathcal{B}} c_b^{\text{staff}} x_{s,b,h}^{\text{staff}}) \tag{8}$$

subject to:

$$\sum_{h\in\mathcal{H}} x_{s,h}^{\text{bed}} \geq D_{s,r} \qquad \forall s \in \mathcal{S}, r \in \mathcal{R} \tag{9}$$

$$\sum_{b'\in\mathcal{B}:b'\geq b} x_{s,b',h}^{\text{staff}} \geq R_{s,b} \cdot x_{s,h}^{\text{bed}} \qquad \forall s \in \mathcal{S}, b \in \mathcal{B}, h \in \mathcal{H} \tag{10}$$

$$x_{s,h}^{\text{bed}} \leq K_{s,h} \qquad \forall s \in \mathcal{S}, h \in \mathcal{H} \tag{11}$$

$$0 \leq \sum_{s\in\mathcal{S}} x_{s,h}^{\text{bed}} \leq UB_h^{\text{max, bed}} \qquad \forall h \in \mathcal{H} \tag{12}$$

$$0 \leq \sum_{s\in\mathcal{S}} \sum_{h\in\mathcal{H}} x_{s,b,h}^{\text{staff}} \leq UB_b^{\text{max, staff}} \qquad \forall b \in \mathcal{B} \tag{13}$$

The cost of staffing and bed deployment in each specialty and facility is kept to a minimum by objective function (8). Restrictions (9) guarantee that the quantity of beds placed meets the needs. Constraint (10) ensure that minimum no. of personnel assigned to each specialization per hospital is met. Limitations (11)

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make sure that the number of beds deployed doesn't go beyond each hospital's maximum number of specialty beds. The decision factors and their domain are indicated in constraints (12) – (13).

Similarly, the **"two-stage stochastic model"** can be formulated as follows:

$$\min \sum_{h\in\mathcal{H}}\sum_{s\in\mathcal{S}}(c_{s,h}^{\text{bed, 1st}} x_{s,h}^{\text{bed}} + \sum_{b\in\mathcal{B}} c_b^{\text{staff, 1st}} x_{s,b,h}^{\text{staff}}) + \sum_{k\in\mathcal{K}}\sum_{h\in\mathcal{H}}\sum_{s\in\mathcal{S}} p_k (c_{s,h}^{\text{bed, 2nd}} u_{s,h,k}^{\text{bed}} + \sum_{b\in\mathcal{B}} c_b^{\text{staff, 2nd}} u_{s,b,k,h}^{\text{staff}}) \quad (14)$$

subject to:

$$\sum_{h\in\mathcal{H}}(x_{s,h}^{\text{bed}} + u_{s,h,k}^{\text{bed}}) \geq D_{s,r,k} \quad \forall s\in\mathcal{S}, r\in\mathcal{R}, k\in\mathcal{K} \quad (15)$$

$$\sum_{b'\in\mathcal{B}:b'\geq b} x_{s,b',h}^{\text{staff}} \geq R_{s,b}\cdot x_{s,h}^{\text{bed}} \quad \forall s\in\mathcal{S}, b\in\mathcal{B}, h\in\mathcal{H} \quad (16)$$

$$\sum_{b'\in\mathcal{B}:b'\geq b} u_{s,b',k,h}^{\text{staff}} \geq R_{s,b}\cdot u_{s,h,k}^{\text{bed}} \quad \forall s\in\mathcal{S}, b\in\mathcal{B}, h\in\mathcal{H}, k\in\mathcal{K} \quad (17)$$

$$x_{s,h}^{\text{bed}} \leq K_{s,h} \quad \forall s\in\mathcal{S}, h\in\mathcal{H} \quad (18)$$

$$u_{s,h,k}^{\text{bed}} \leq K_{s,h} \quad \forall s\in\mathcal{S}, h\in\mathcal{H}, k\in\mathcal{K} \quad (19)$$

$$0 \leq \sum_{s\in\mathcal{S}} x_{s,h}^{\text{bed}} \leq UB_h^{\text{max, bed, 1st}} \quad \forall h\in\mathcal{H} \quad (20)$$

$$0 \leq \sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}} x_{s,b,h}^{\text{staff}} \leq UB_b^{\text{max, staff, 1st}} \quad \forall b\in\mathcal{B} \quad (21)$$

$$0 \leq \sum_{s\in\mathcal{S}} u_{s,h,k}^{\text{bed}} \leq UB_h^{\text{max, bed, 2nd}} \quad \forall h\in\mathcal{H}, k\in\mathcal{K} \quad (22)$$

$$0 \leq \sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}} u_{s,b,k,h}^{\text{staff}} \leq UB_b^{\text{max, staff, 2nd}} \quad \forall b\in\mathcal{B}, k\in\mathcal{K} \quad (23)$$

As mentioned, the first summation in the objective function (14) represents the costs associated with assigning beds and allocating personnel to specialties in each designated institution. The second summation captures the costs of deploying additional resources, either within the same hospital or in another facility in the region. The first limitation (15) ensures that quantity of hospital beds deployed meets the demand for each specialty and location. The scenario parameter affects the demand. While Constraints (17) guarantee that this criterion is fulfilled in the second stage, Constraints (16) guarantee that the quantity of staff deployed in the first stage satisfies the minimal criteria for staff on each specialized ward. The beds deployed are kept within the maximum bed capacity per specialty inside each institution by the constraints (18) and (19). The domains and decision factors are indicated by constraints (20)-(23).

**Evaluation of Measures:** It is well known in prescriptive analytics that the EV solution may exhibit suboptimal behavior in the stochastic domain. To ascertain the robustness and performance of each EV, RP, and EEV, conventional evaluation tests can be conducted. "**Maggioni and Wallace (2010)"** suggested few tests to assess limit to 4 for **stochastic model's** performance evaluation metrics. For this study, the initial

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approach for determining the value of the stochastic solution (VSS) will be used. Let $\bar{x}(\bar{\xi})$ denote the optimal solution to Equation (7). After assigning and fixing values for the first stage, the second stage of the stochastic model can be carried out.

$$EEV = E_{\xi}(z(\bar{x}(\bar{\xi}), \xi))$$

By evaluating the predicted gain in value from solving the stochastic model as opposed to the simpler deterministic model, the difference between the Expected Economic Value (EEV) and the Risk Profile (RP) can be calculated to determine the Value of the Stochastic Solution (VSS).

$$VSS = EEV - RP$$

When employing the deterministic solution, the VSS calculates the expected loss. The estimated cost of the deterministic solution is frequently ∞ if we have strong limitations. However, by using the deterministic technique to set penalties high, we can employ soft constraints to make the projected cost arbitrarily awful. A significant VSS may indicate that the variables were not selected correctly or that the values were input incorrectly.

**Illustrative Example:** We provide a practical example with fictitious numerical data to demonstrate the r elevance of our suggested paradigm.
The optimization procedure and important results are demonstrated in this example.
In order to give a clear example of the stochastic programming approach while preserving computational t raceability, a small number of situations are added in the second stage.
A dataset of 11 patients could be used to demonstrate how well the model works in caring for elderly and fragile patients.
The dataset in the table below comprises two hospitals in the same region that offer services for the same two specialties: orthopedics and trauma (T&O) and care of the elderly (COTE).
We assume that the wards need two nursing staff band levels, with different staff/bed ratios depending on the specialism.

| Patient Number | Age | Hospital | LOS | Specialty | Admission Method | Admission Source | Frailty Source |
|---|---|---|---|---|---|---|---|
| Patient 1 | 95 | 1 | 5 | COTE | Emergency | Own Home | 3 |
| Patient 2 | 82 | 1 | 3 | COTE | Emergency | Own Home | 2 |
| Patient 3 | 89 | 1 | 4 | T&O | Emergency | Own Home | 2 |
| Patient 4 | 87 | 1 | 4 | T&O | Elective | Own Home | 2 |
| Patient 5 | 85 | 2 | 3 | COTE | Elective | Transferred | 1 |
| Patient 6 | 76 | 2 | 1 | COTE | Elective | Transferred | 1 |
| Patient 7 | 71 | 2 | 1 | T&O | Emergency | Transferred | 1 |
| Patient 8 | 96 | 1 | 5 | T&O | Emergency | Own Home | 3 |
| Patient 9 | 70 | 2 | 1 | COTE | Emergency | Transferred | 1 |
| Patient 10 | 67 | 2 | 1 | T&O | Elective | Own Home | 1 |

***Table 4: Worked Example Patient Data***

Table 5 shows the parameters and corresponding values for the deterministic and stochastic models. The demand from Table 4 can be used to calculate the average daily bed demand.

$$\text{Average daily bed demand}_{s,h} = \text{Average LOS}_{s,h} \times \text{Average daily number of admissions}_{s,h}$$

$$D_{s,r} = \text{Average daily bed demand}_{s,r} = \sum \text{Average daily bed demand}_{s,h}$$

This results in values of 16.67 for the parameter D0,0 and 19.01 for D1,0. The two-stage stochastic model necessitates numerous scenarios to effectively represent the uncertainty involved in the problem.

To demonstrate the model's operation and provide an example, we present three situations that add up to t he same deterministic demand.: There is a 40% probability that demand will remain at the average level, a 30% chance that it will decrease by 20%, and a 30% chance that it will increase by 20%. The demand matrix **Ds,r,k** can therefore be displayed as follows**: [[16.66, 19.99, 13.33],[19.01, 22.80, 15.20]]**

The first index corresponds to the row location, while the second corresponds to the column. Since we're dealing with only one region, the matrix has just a single column.

| **Parameters** | **Values** | **Parameters** | **Values** |
|---|---|---|---|
| $1^{st}$ Stage Bed Costs ($c_{s,h}^{\text{bed,1st}}$) | $\begin{bmatrix} 20 & 30 \\ 30 & 40 \end{bmatrix}$ | $2^{nd}$ Stage Bed Costs ($c_{s,h}^{\text{bed,2nd}}$) | $\begin{bmatrix} 22 & 33 \\ 33 & 44 \end{bmatrix}$ |
| Ratio ($R_{s,b}$) | $\begin{bmatrix} 0.29 & 0.14 \\ 0.14 & 0.29 \end{bmatrix}$ | Maximum Specialty Capacity ($K_{s,h}$) | $\begin{bmatrix} 20 & 25 \\ 20 & 25 \end{bmatrix}$ |
| $1^{st}$ Stage Staff Costs ($c_b^{\text{staff, 1st}}$) | [£50, £60] | $2^{nd}$ Stage Staff Costs ($c_b^{\text{staff, 2nd}}$) | [£55, £66] |
| Upper $1^{st}$ bed limit ($UB_h^{\text{max,bed,1st}}$) | [20,25] | Upper $2^{nd}$ bed limit ($UB_{h,k}^{\text{max,bed,2nd}}$) | $\begin{bmatrix} 20 & 20 & 20 \\ 25 & 25 & 25 \end{bmatrix}$ |
| Upper $1^{st}$ staff limit ($UB_b^{\text{max,staff,1st}}$) | [15,25] | Upper $2^{nd}$ staff limit ($UB_{b,k}^{\text{max,staff,2nd}}$) | $\begin{bmatrix} 15 & 15 \\ 25 & 25 \end{bmatrix}$ |
| Probability of Scenarios ($p_k$) | [0.4,0.3,0.3] | | |

**Table 5 The parameter values that were used within the deterministic and two-stage stochastic model specifically for the illustrative example.**

The column within the sub-matrix is referred to by the third index. The ideal deciding variables and objective function values for the worked case are mentioned below.

| | s=0, h=0 | s=0, h=1 | s=1, h=0 | s=1, h=1 | Objective Value (£) |
|---|---|---|---|---|---|
| **EV** | [(0), (0,0)] | [(17), (5,3)] | [(20), (3,6)] | [(0), (0,0)] | 2,050.00 |
| **RP** | [(20), (6,3)] | [(1), (1,1)] | [(24), (4,8)] | [(0), (0,0)] | 2,185.20 |
| **EEV** | [(4), (2,1)] | [(17), (5,3)] | [(24), (4,8)] | [(0), (0,0)] | 2,240.80 |

Table 6 Deterministic and Two-Stage Stochastic Results for the Worked Example - Results are Recorded [(beds), (staff)]

The result set shows that deterministic model produces an expected value (EV) solution that is about one-third less expensive than the robust (RP) solution, as it utilizes fewer beds and less nursing staff compared to the stochastic method. The Expected Value of the Expected Value (EEV) is measured by taking the optimal **"first-stage decision variables from the deterministic model**" and applying them within the **"two-stage stochastic"** framework. Next, the values for the second-stage variables and the objective function are established. The difference between the EEV and RP is then used to calculate the VSS, which comes out to £55.60, or a 2.54% savings, if the stochastic solution were to be used. Because the EEV exceeds the RP, the results also demonstrate the deterministic model's lack of robustness. This is because the demand's uncertainty cannot be taken into account by the deterministic model.

## FUTURE RESEARCH DIRECTION

- ***Improved Integration***: More research into combining "predictive and prescriptive models" to solve operational issues in the medical field.
- ***Real-World Data Application***: Creating models that are capable of efficiently capturing and applying patient data variations found in the real world.
- ***Ethical Frameworks***: Creating rules to handle moral issues with "predictive and prescriptive analytics" in safe settings for easier and safe utilization.

## KEY FINDINGS AND TRENDS

- ***Integration of Predictive and Prescriptive Analytics***: It has been demonstrated that combining customized therapies with predictive models enhances clinical results in older persons, assisting in the management of population health.
- ***Machine Learning Advancements***: Using machine learning techniques has improved the precision of forecasting unfavorable health occurrences, allowing for early interventions.

## SUMMARY

This paper presents an approach to improving healthcare services by integrating predictive analytics using CART models with prescriptive analytics through deterministic and two-stage stochastic optimization. The research addresses key challenges in resource allocation across multiple healthcare sites by combining data-driven predictions with robust decision-making frameworks. Through a worked example, the model demonstrates how predictive insights, such as patient length of stay, can be directly linked to optimized

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resource planning, including staffing and bed deployment. The fusion of these analytics methods results in more accurate, efficient, and adaptable healthcare delivery, especially under demand uncertainty.

## CONCLUSION

This research work demonstrates the outcome of integrating "predictive and prescriptive analytics" to improve medical service delivery, especially in multi-site hospital networks. Predictive models such as CART effectively anticipate patient needs, while two-stage stochastic programming ensures optimal resource allocation amid uncertainty. The proposed framework offers a robust, scalable, and data-driven approach to enhancing patient care and operational efficiency. The illustrative example highlights the practical advantages of this integrated methodology. Future research should aim to expand real-world applications, incorporate ethical considerations, and refine model adaptability using diverse datasets.

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