Differential ideals in a Ternary semigroup

Jyothi.Gaddipati [1], M.Dhanalakshmi[2] ,K.Bhanu Priya

[1,2,]Associate Professor Dhanekula Institute of Engineering &Technology,

[3] Assistant Professor sri durga malleswara Siddhartha mahila kalasala.

Abstract: In this paper we introduced the notion of derivation in a ternary semigroup. We observe that the set of all differential ideals forms a complete lattice with respect to set inclusion relation. Finally, we deduced that in a differential ternary semigroup satisfying ascending chain condition on radical differential ideals any radical differential ideal is expressible as the intersection of finite number of differential ideals.

Keywords: Differential ternary semigroup, d-differential ideal, differential k-ideal, Radical differential ideal.

1.Introduction:

The notions of groups with derivations is quite old and plays a significant role in the integration of analysis, algebraic geometry and algebra. The fundamental and important relations between the operation of differentiation and that of addition and multiplication of functions have been known for as long a time as the notion of the derivative itself. We prove that Let S be a ternary differential semigroup and I be a ideal of S. Then the Bourne factor ternary semigroup S/I is a differential ternary semigroup.The notion of derivation has also been generalized in various directions, such as Jordan derivation, generalized derivation, generalized Jordan derivation etc. Jonathan Golan mentioned about the derivation on a semiring. This motivated some authors to study about the properties of derivations in a semigroup as well as in a Γ-semigroup.

2. **Derivation in a Ternary Semigroup**

Throughout this paper unless otherwise stated a ternary semigroup means commutative ternary semigroup with zero

Definition 2.1.: Let S be a ternary semigroup. A mapping d : S → S is said to be a derivation on S if (i) $(a_1 + a_2)' = a'_1 + a'_2$ for all $a_1, a_2 \in S$, (ii) $(a_1a_2a_3)' = a'_1a_2a_3 + a_1a'_2a_3 + a_1a_2a'_3$ for all $a_1, a_2, a_3 \in S$, (iii) $0' = 0$, (iv) $e' = 0$, provided S has unital element e. We call d(a), that is a′ the derivative of a.

Note 2.2: For every ternary semigroup S there exists a derivation d on S, viz, d(s) = 0 for all $s \in S$. This derivation is called the trivial derivation.

Definition 2.3: A ternary semigroup together with a non -trivial derivation is said to be a differential ternary semigroup.

Proposition 2.4: Let S be a ternary semigroup and F denote the set of all derivations on S. We define (D1 +D2)(x) = D1(x) +D2(x) for $x \in S$ and D1,D2 $\in$ F. Then (F,+) is a monoid.

Theorem 2.5: In any differential ternary semigroup S, the elements with derivative zero forms a ternary subsemigroup of S.

Proof: let $C = \{x \in S : x' = 0\}$. Clearly $0 \in C$. So C is nonempty. Let $a, b, c \in C$. Then $a' = 0$, $b' = 0$, $c' = 0$. Now $(a+b)' = a' + b' = 0+0 = 0$. and $(abc)' = a'bc + ab'c + abc' = 0bc + a0c + ab0 = 0+0+0 = 0$. Thus $a+b \in C$ and $abc \in C$. Hence C is a ternary subsemigroup of S.

3.Differential Ideals in a Ternary Semigroup

Definition 3.1: If d is a nontrivial derivation on a ternary semigroup S then an ideal I of S is said to be a d-differential ideal of S if a' □ I whenever a □ I.

We simply write differential ideal instead of d-differential ideal.

Definition 3.2: A k-ideal (h-ideal) which is also a differential ideal is said to be a differential k-ideal (h-ideal).

Theorem 3.3: In a differential ternary semigroup intersection of any collection of differential ideals is again a differential ideal**.**

Definition 3.4: An equivalence relation ρ on a ternary semigroup S is said to be a congruence on S if the following conditions hold: (i) $a\rho a_1$ and $b\rho b_1$ □ $(a + b)\rho(a_1 + b_1)$ (ii) $a\rho a_1, b\rho b_1, c\rho c_1$ □ $(abc)\rho(a_1b_1c_1)$ for all a, b, c, a_1, b_1, c_1 □ S.

Definition 3.5: Let I be a proper ideal of a ternary semigroup S. Then the congruence on S denoted by ρ_1 and defined by $r\rho_1 r_1$ if and only if $r+a_1 = r_1+a_2$ for some a_1,a_2 □ I and r, r_1 □ S in called Bourne congruence on S defined by the ideal I.

Note 3.6: The Bourne congruence class of an element r □ S is denoted by r/ρ_1 or simply by r/I and the set of all such congruence classes of Sis denoted by S/ρ_1 or simply by S/I.

Remark 3.7: It should be noted that for any proper ideal I of S and r/I is not necessarily equal to r + I = {s +a : a □ I} but surely contains it.

Definition 3.8: For any proper ideal I of a ternary semigroup S if the Bourne congruence ρ_1 defined by I is proper then we can define addition and ternary multiplication in S/I by $(a/I + b/I) = (a + b)/I$ and $(a/I)(b/I)(c/I) = (abc)/I$ for all a, b, c □ S. With these two operations S/I forms a ternary semigroup which is called the Bourne factor ternary semigroup or simply the factor ternary semigroup.

Theorem 3.9: Let S be a ternary differential semigroup and I be a ideal of S. Then the Bourne factor ternary semigroup S/I is a differential ternary semigroup.

Proof. We define a mapping $d: S/I \rightarrow S/I$ by $d(r/I) = r'/I$, where r' is the derivative of r *in S*.

Let r_1, r_2 □ S.

Then $d(r_1/I + r_2/I) = d((r_1 + r_2)/I)$

$=(r_1 + r_2)'/I$

$=(r'_1 + r'_2)/I$

$=r'_1/I + r'_2/I$

$=d(r_1/I) + d(r_2/I)$. $d((r_1/I)(r_2/I)(r_3/I)) = d(r_1r_2r_3/I)$

$=(r_1r_2r_3)'/I =(r'_1r_2r_3 + r_1s'_2r_3 + r_1r_2r'_3)/I$

$=(r'_1r_2r_3/I) + (r_1r'_2r_3/I) + (r_1r_2r'_3/I)$

$=(r'_1/I)(r_2/I)(r_3/I) + (r_1/I)(r'_2/I)(r_3/I) + (r_1/I)(r_2/I)(r'_3/I)$

$=d(r_1/I)(r_2/I)(r_3/I) + (r_1/I)d(r_2/I)(r_3/I) + (r_1/I)(r_2/I)d(r_3/I)$.

So d is a derivation on S/I.

Hence S/I is a differential ternary semigroup.

4.Radical Differential Ideals in a Ternary Semigroup

Definition 4.1: A radical ideal (k-ideal, h-ideal) which is also a differential ideal is called a radical differential ideal (k-ideal, h-ideal)

Theorem 4.2: If *abc* lies in a radical differential k-ideal I of a ternary semigroup S, then *a'bc* □ *I,ab'c* □ *I,abc'* □ *I.*

Proof. Let *abc* □ *I*. Then (*abc)'* □ *I* i.e., (a*'bc + ab'c + abc')* □ *I.* Which implies *abc'abc'(a'bc + ab'c + abc')* □ *I* i.e., (*abc')2a'bc + (abc')2ab'c + (abc')3* □ *I.* Now *(abc')2a'bc* □ *I*, (*abc')2ab'c* □ *I*. Hence (*abc')3* □ *I*, as I is a k-ideal. *abc'* □ *I*. Similarly we can prove *a'bc* □ *I, ab'c* □ *I.*

Definition 4.3: A nonempty subset A of ternary semigroup S is called an m- system if for each *a,b,c* □ *A* there exist elements x_1,x_2,x_3,x_4 of S such that ax_1bx_2c □ *A* or $ax_1x_2bx_3x_4c$ □ *A* or $ax_1x_2bx_3cx_4$ □ *A or* $x_1ax_2bx_3x_4c$ □ *A.*

Definition 4.4: Let S be a ternary semigroup and A be any subset of S. The differential ideal generated by A is denoted by {A} and defined to be intersection of all differential ideals of S which contains A.

For simplicity we write {A □ *{a}} as {A, a}.*

Theorem 4.5: Let P be an m-system in a differential ternary semigroup S and I be differential ideal of S which does not meet P. Then I is contained in a maximal differential ideal Q of S which does not meet P and such Q is prime.

Proof. Let U be the set of all differential ideals of S containing I and none of which meets P. Then U is a po-set with respect to set inclusion relation. Then Zorn's lemma ensures that U has a maximal element Q (say). Therefore P ∩ Q = φ.

If possible let Q is not prime, then there exists *a,b,c* □ *S* such that *a* / □ *Q, b* / □ *Q, c* / □ *Q* but *abc* □ *Q*. Then *{Q,a}, {Q,b}, {Q,c}* are differential ideals properly containing Q. Then by maximality of Q, *{Q,a}∩P= φ*, *{Q,b}∩P= φ* and *{Q,c}∩P= φ.*

Let p_1 □ *{Q,a}∩P,* p_2 □ *{Q,b}∩P,* p_3 □ *{Q,c}∩P*. Since *P* is an *m-system* there exist r_1,r_2,r_3,r_4 □ *S* such that $p_1r_1p_2r_2p_3$ □ *P* or $p_1r_1r_2p_2r_3r_4p_3$ □ *P* or $p_1r_1r_2p_2r_3p_3r_4$ □ *P or* $r_1p_1r_2p_2r_3r_4p_3$ □ *P.*

If $p_1r_1p_2r_2p_3$ □ P then $p_1r_1p_2r_2p_3$ □ $\{Q,a\}\{Q,b\}\{Q,c\}$ □ $\{Q, abc\}$ □ Q, which contradicts the fact that $P \cap Q = \varphi$.

If $p_1r_1r_2p_2r_3r_4p_3$ □ P then $p_1r_1r_2p_2r_3r_4p_3$□ $\{Q,a\}\{Q,b\}\{Q,c\}$ □ $\{Q,abc\}$ □ Q, which contradicts the fact that $P \cap Q = \varphi$. *If* $p_1r_1r_2p2r3p3r4$ □ *P then* $p1r1r2p2r3p3r4$ □ $\{Q,a\}\{Q,b\}\{Q,c\}$ □ $\{Q,abc\}$ □ Q, which contradicts the fact that P ∩ Q = φ. If r1p1r2p2r3r4p3 □ P then r1p1r2p2r3r4p3 □ {Q,a}{Q,b}{Q,c} □ {Q,abc} □ Q, which contradicts the fact that P ∩Q = φ. Thus in any case we arrive at a contradiction. Consequently, Q is prime

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