PERISTALTIC TRANSPORT OF COUPLE STRESS FLUID THROUGH A POROUS MEDIUM WITH VARIABLE VISCOSITY AND AN ENDOSCOPE

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Abstract

The peristaltic transport of couple stress fluid through a porous media with variable viscosity in the gap between coaxial tubes has been studied. Where outer tube is non-uniform sinusoidal wave traveling down in wall and inner tube is rigid. The relation between viscosity, pressure gradient and friction force on inner and outer tubes have been obtained in terms of couple stress parameter. The numerical solution of pressure gradient, outer friction, inner friction and flow rate are shown graphically.

Keywords: Peristaltic transport, Couple Stress fluid, Porous media, Viscosity parameter.

1. INTRODUCTION

The aim of this paper is to study a physiological situation with the presence of an endoscope placed concentrically. Srivastava et. al., (1) have investigated peristaltic transport of a physiological fluid: Part I Flow in Non- Uniform Geometry. Latham (2) investigated fluid motion in a peristaltic pump. Ramachandra and Usha (3) investigated peristaltic transport of two immiscible viscous fluids in a circular tube. Gupta and Sheshadri (4) investigated peristaltic pumping in non-uniform tubes. Cotton and Williams (5) have investigated practical gastrointestinal endoscopy. Mekhemier (6) investigated non linear peristaltic transport a porous medium in an Inclined Planar Channel. Srivastava and Srivastava (7) investigated peristaltic transport of a non-newtonian fluid: applications to the vas deferens and small intestine. Mekhemier and Abd elmaboud (8) investigated peristaltic flow of a couple stress fluid in an annulus : Application of an endoscope. Srivastava et.al., (9) investigated peristaltic transport of a Physiological Fluid: Part I Flow in Non- Uniform Geometry. abd elmaboud and mekheimer (10) investigated non-linear peristaltic transport of a second-order fluid through a porous medium. Cotton and Williams (11) have investigated practical Gastrointestinal Endoscopy. Ramachandra and Usha (12) investigated peristaltic transport of Two Immiscible Viscous Fluids in a Circular Tube. Raptis and Peridikis (13) investigated flow of a viscous fluid through a porous medium bounded by a vertical surface. Rathod et al. (14-19) investigated peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium, non - erodible porous lining tube wall with thickness of porous material, through a porous medium in a channel. Sridhar (20-23) investigated effect of thickness of the porous material using porous media on the peristaltic pumping of couple stress fluid through non - erodible porous lining tube and fluid in an inclined channel under the effect of magnetic field through a porous medium with slip condition.

We propose to study peristaltic transport of a viscous incompressible fluid (creeping flow) through gap between coaxial tubes, where outer tube is non-uniform and has a sinusoidal wave traveling down its wall and inner one is rigid, uniform tube and moving with a constant velocity. This investigation may have clinical applications such as endoscopes problem. In this paper, Couple Stress Fluid with Variable Viscosity and an Endoscope on Peristaltic Transport through a Porous Media is investigated.

2. FORMULATION OF THE PROBLEM

Consider the peristaltic flow of an couple stress fluid through coaxial tubes such that the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is rigid, uniform and moving with a constant velocity. The geometry of the two wall surfaces are:

$$r_1^* = a_1 \quad (1)$$

$$r_2^* = a_{20} + bSin(\frac{2\pi}{\lambda}(z^* - ct^*)) \quad (2)$$

Where a_1 is the radius of endoscope tube, a_{20} is radius of the small intestine at inlet, b is the wave amplitude, λ is the wavelength, t is time and c is the wave speed.

We choose cylindrical coordinate system (r^*, z^*), the flow in the gap between inner and outer tube is unsteady but if we choose moving coordinates (r^*, z^*) which travel in the z-axis lies along centerline of inner and outer tubes and r^* is distance measured racially. The coordinate frames are related through

$$z^* = Z^* - ct^*, \; r^* = R^*, \quad (3)$$
$$w^* = W^* - c, \; u^* = U^*$$

Where, U^*, W^* and u^*, w^* are the velocity components in the radial and axial direction in the fixed and moving coordinates, respectively.

The Navier - Stokes equations are:

$$\frac{1}{r^*}\frac{\partial(r^*,u^*)}{\partial r^*}+\frac{\partial(w^*)}{\partial z^*}=0 \quad (4)$$

$$\rho\{u^*\frac{\partial u^*}{\partial r^*}+w^*\frac{\partial u^*}{\partial z^*}\}=-\frac{\partial p^*}{\partial r^*}+\frac{\partial}{\partial r^*}[2\mu^*(r^*)\frac{\partial u^*}{\partial r^*}]+2\frac{\partial\mu^*(r^*)}{r^*}(\frac{\partial u^*}{\partial r^*}-\frac{u^*}{r^*})+\frac{\partial}{\partial z^*}[\mu^*(r^*)(\frac{\partial u^*}{\partial z^*}+\frac{\partial w^*}{\partial r^*})]-\eta\nabla^2(\nabla^2(u^*))-\frac{\mu}{K}u^* \quad (5)$$

$$\rho\{u^*\frac{\partial w^*}{\partial r^*}+w^*\frac{\partial w^*}{\partial z^*}\}=-\frac{\partial p^*}{\partial z^*}+\frac{\partial}{\partial z^*}[2\mu^*(r^*)\frac{\partial w^*}{\partial z^*}]+\frac{1}{r^*}\frac{\partial}{\partial r^*}[\mu^*(r^*)r^*(\frac{\partial u^*}{\partial z^*}+\frac{\partial w^*}{\partial r^*})]-\eta\nabla^2(\nabla^2(w^*))-\frac{\mu}{K}w^* \quad (6)$$

Where, $\nabla^2=\frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*\frac{\partial}{\partial r^*})$

Where p^* is pressure, $\mu^*(r^*)$ is the viscosity function, ρ is density, μ is viscosity, η is couple stress parameter and K is porous media.

The boundary conditions are:

$$\begin{aligned} &w^*=-c, \quad \nabla^2(w^*) \quad finite \quad at \quad r^*=r_1^* \\ &u^*=0, \nabla^2(w^*)=0 \quad at \quad r^*=r_2^* \end{aligned} \quad (7)$$

We introduce the non-dimensional variable and Reynolds number (Re) and wave number (δ) introduced:

$$\begin{aligned} &z=\frac{z^*}{\lambda^*},\ Z=\frac{Z^*}{\lambda^*},R=\frac{R^*}{a_{20}},\mu(r)=\frac{\mu^*(r^*)}{\mu},u=\frac{\lambda u^*}{a_{20}c},\ U=\frac{\lambda U^*}{a_{20}c},\ p=\frac{a_{20}^2}{\lambda\mu c}p^*(z^*),\ t=\frac{t^*c}{\lambda}, \\ &\mathrm{Re}=\frac{\rho c a_{20}}{\mu},w=\frac{w^*}{c},W=\frac{W^*}{c},\eta=l^2\rho\gamma,\delta=\frac{a_{20}}{\lambda}<<1,K=\frac{K^*}{\lambda},\ r_1=\frac{r_1^*}{a_{20}}=\varepsilon<1, \\ &r_2=\frac{r_2^*}{a_{20}}=1+\phi\sin 2\pi z,\phi=\frac{b}{a_{20}}<1 \end{aligned} \quad (8)$$

Where, ε is the radius ratio, ϕ is the amplitude ratio, μ is the viscosity on the endoscope.

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \quad (9)$$

$$\mathrm{Re}\,\delta^3\{u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\}=-\frac{\partial p}{\partial r}+\delta^2\frac{\partial}{\partial r}(2\mu(r)\frac{\partial u}{\partial r})+2\delta^2\frac{\mu(r)}{r}(\frac{\partial u}{\partial r}-\frac{u}{r})+\delta^2\frac{\partial}{\partial z}[\mu(r)(\delta^2\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r})]-\frac{\delta^2}{\gamma^2}\nabla^2(\nabla^2(u))-\delta^2h^2(u) \quad (10)$$

$$\mathrm{Re}\,\delta\{u\frac{\partial u}{\partial r}+w\frac{\partial w}{\partial z}\}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}[\mu(r).r.(\delta^2\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r})]+\delta^2\frac{\partial}{\partial z}(2\mu(r)\frac{\partial w}{\partial z})-\frac{1}{\gamma^2}\nabla^2(\nabla^2(w))-h^2(w) \quad (11)$$

Where, $\gamma=\sqrt{\frac{\eta}{\mu a_{20}^2}}$ couple-stress parameter & $h=\sqrt{\frac{a_{20}}{K}}$ porous media.

The dimensionless boundary conditions are:

w = -1 , $\nabla^2(u,w)$ finite at r = r_1 (12)

u = 0 , $\nabla^2(u,w) = 0$ finite at r = r_2

Using long wavelength approximation and neglecting the wave number δ, Navier Stokes equations reduces to:

$$\frac{\partial p}{\partial r} = 0 \tag{13}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}[\mu(r).r.\frac{\partial w}{\partial r}] - \frac{1}{\gamma^2}\nabla^2(\nabla^2(w)) - h^2(w) \tag{14}$$

The instantaneous volume flow rate in the fixed coordinate system is given by

$$Q^* = \int_{r_1^*}^{r_2^*} 2\pi R^* W^* dR^* \tag{15}$$

Where, r_1^* is a constant and r_2^* is a function of z^* and t^*. On substituting (3) into (15) and on integrating, we obtain

$$Q^* = q^* + \pi c(r_2^{*2} - r_1^{*2}) \tag{16}$$

Where, $$q^* = \int_{r_1^*}^{r_2^*} 2\pi r^* w^* dr^* \tag{17}$$

is the volume flow rate in the moving coordinate system and is it independent of time? Here, r_2^* is a function of z^* alone and is defined through (2). Using the dimensionless variable, we find that (17) becomes

$$F = \frac{q^*}{2\pi a_{20}^2 c} = \int_{r_1}^{r_2} rwdr \tag{18}$$

The time-mean flow over a period $T = \lambda/c$ at a fixed Z position is defined as

$$Q = \frac{1}{T} = \int_0^T Q^* dt^* \tag{19}$$

Using (16) and (17) in (19) and integrating, we get

$$Q^* = q^* + \pi c(a_2^2 - a_1^2 + \frac{b^2}{2}) \tag{20}$$

which may be written as

$$\frac{Q^*}{2\pi a_{20}^2 c} = \frac{q^*}{2\pi a_{20}^2 c} + \frac{1}{2}(1-\varepsilon^2+\frac{\phi^2}{2}) \qquad (21)$$

On defining the dimensionless time-mean flow as

$$\Theta = \frac{Q^*}{2\pi a_{20}^2 c} \qquad (22)$$

Writing (21) as

$$\Theta = F + \frac{1}{2}(1-\varepsilon^2+\frac{\phi^2}{2}) \qquad (23)$$

and using boundary conditions (12) to eqns.(13) & (14), we obtain

$$w = \frac{1}{2}\frac{\partial p}{\partial z}[X_1 - \frac{1}{8\gamma^2}X_2.X_3 + \frac{2}{K^2}] + \frac{1}{4\gamma^2}X_2 \qquad (24)$$

Where,

$$X_1 = I_1(r) - I_1(r_1) + \frac{I_1(r_1)-I_1(r_2)}{I_2(r_2)-I_2(r_1)}\{I_2(r)-I_2(r_1)\},\ X_2 = r^2 - r_1^2 + (r_1^2 - r_2^2)(\frac{In(\frac{r}{r_1})}{In(\frac{r_2}{r_1})}),\ X_3 = r^2 - r_1^2 + 2(r_1^2 - r_2^2)(\frac{In(\frac{r}{r_1})}{In(\frac{r_2}{r_1})})$$

$$I_1(r) = \int \frac{r}{\mu(r)}dr,\ I_2(r) = \int \frac{dr}{r.\mu(r)} \qquad (25)$$

using (18) we obtain the relationship between dp/dz and F as follows:

$$F = \frac{1}{2}\frac{dp}{dz}[I_3 - I_1(r_1).\frac{(r_2^2-r_1^2)}{2} + \frac{I_1(r_1)-I_1(r_2)}{I_2(r_2)-I_2(r_1)}\{I_4 - I_2(r_1)\frac{(r_2^2-r_1^2)}{2}\} - \frac{1}{8\gamma^2}\{\frac{(r_2^6-r_1^6)}{6} + \frac{r_1^2.r_2^2.(r_1^2-r_2^2)}{2} + \frac{3.(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{Z_1\} - \frac{(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{r_1^2 - \frac{4.(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\}.\{Z_2\} + \frac{(r_2^2-r_1^2)}{K^2}\}] + \frac{1}{4\gamma^2}[\frac{(r_2^4-r_1^4)}{4} - \frac{r_1^2.(r_2^2-r_1^2)}{2} + \frac{(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{Z_2\}] \qquad (26)$$

Where, $$Z_1 = \frac{(-1+4.In(r_1)).r_1^4+(1-4In(r_2)).r_2^4}{16(r_1-r_2)} - \frac{In(r_1).(r_2^4-r_1^4)}{4},\qquad I_3 = \int_{r_1}^{r_2}\frac{r^2}{\mu(r)}dr,$$

$$Z_2 = \frac{(-1+2.In(r_1)).r_1^2+(1-2.In(r_2)).r_2^2}{4.(r_1-r_2)} - \frac{In(r_1).(r_2^2-r_1^2)}{2},\qquad I_4 = \int_{r_1}^{r_2}\frac{dr}{\mu(r)} \qquad (27)$$

Solving (26) for dp/dz, we obtain

$$\frac{dp}{dz}=\frac{2F-\frac{1}{2\gamma^2}[\frac{(r_2^4-r_1^4)}{4}-\frac{r_1^2.(r_2^2-r_1^2)}{2}+\frac{(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{Z_2\}]}{[I_3-I_1(r_1).\frac{(r_2^2-r_1^2)}{2}+\frac{I_1(r_1)-I_1(r_2)}{I_2(r_2)-I_2(r_1)}\{I_4-I_2(r_1)\frac{(r_2^2-r_1^2)}{2}\}-\frac{1}{8\gamma^2}\{\frac{(r_2^6-r_1^6)}{6}+\frac{r_1^2.r_2^2.(r_1^2-r_2^2)}{2}+\frac{3.(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{Z_1\}-\frac{(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\{r_1^2-\frac{4.(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}\}.\{Z_2\}+\frac{(r_2^2-r_1^2)}{K^2}\}]} \tag{28}$$

The pressure rise Δp_λ and friction force (at the wall) on outer and inner tubes $F_\lambda^{(o)}$ *and* $F_\lambda^{(i)}$ respectively, in their non-dimensional forms, are given by

$$\Delta P_\lambda=\int_0^1 (\frac{dp}{dz})dz, F_\lambda^{(o)}=\int_0^1 r_2^2(-\frac{dp}{dz})dz, F_\lambda^{(i)}=\int_0^1 r_1^2(-\frac{dp}{dz})dz \tag{29}$$

The effect of viscosity variation on peristaltic transport can be investigated through (29) for any given viscosity function $\mu(r)$.

For the present investigation, we assume viscosity variation in the dimensionless form following Srivastava et al. (1) as follows:

$$\mu(r)=e^{-ar} \tag{30}$$

Or $$\mu(r)=1-\alpha r \quad \text{for} \quad \alpha \ll 1 \tag{31}$$

where, α is viscosity parameter. The assumption is reasonable for the following physiological reason. Since a normal person of animal or similar size takes 1 to 2 L of fluid every day, another 6 to 7 L of fluid are received by the small intestine daily as secretion from salivary glands, stomach, pancreas, liver, and the small intestine itself. This implies that concentration of fluid is dependent on the radial distance. Therefore, the above choice of $\mu(r)=e^{-ar}$ is justified.

Substituting (31) into (25) & (27), and using (28), we obtain

$$
\begin{aligned}
&\frac{dp}{dz}=[\{2\theta-(1-\varepsilon^2+\frac{\phi^2}{2})-\frac{1}{2\gamma^2}(\frac{(r_2^4-r_1^4)}{4}-\frac{r_1^2.(r_2^2-r_1^2)}{2}+\frac{(r_1^2-r_2^2)}{In(\frac{r_2}{r_1})}(\frac{(-1+2In(r_1)).r_1^2+(1-2In(r_2)).r_2^2}{4.(r_1-r_2)}-\frac{In(r_1).(r_2^2-r_1^2)}{2}))\}/\\
&\{(-\frac{2\alpha+\frac{2(In(-1+\alpha.r_1)-In(-1+\alpha.r_2))}{(r_1-r_2)}+\alpha^2(r_1+r_2)}{2\alpha^3})-(-\frac{r_1}{\alpha}-\frac{In(-1+\alpha.r_1)}{\alpha^2})(\frac{(r_2^2-r_1^2)}{2})+(((-\frac{r_1}{\alpha}-\frac{In(-1+\alpha.r)}{\alpha^2})-(-\frac{r_2}{\alpha}-\\
&\frac{In(-1+\alpha.r_2)}{\alpha^2})(\frac{-In(1-\alpha.r_1)+In(1-\alpha.r_2)}{\alpha(r_1-r_2)}-(-In(-1+\alpha.r_1)+In(r_1))\frac{(r_2^2-r_1^2)}{2}))/((-In(-1+\alpha.r_2)+In(r_2))\\
&-(-In(-1+\alpha.r_1)+In(r_1))))-\frac{1}{8\gamma^2}(\frac{(r_2^6-r_1^6)}{6}+\frac{r_1^2.r_2^2.(r_1^2-r_2^2)}{2}-\frac{3(r_2^2-r_1^2)}{In(\frac{r_2}{r_1})}(\frac{(-1+4In(r_1)).r_1^4+(1-4In(r_2)).r_2^4}{16.(r_1-r_2)}-\\
&\frac{In(r_1).(r_2^4-r_1^4)}{4})+\frac{(r_2^2-r_1^2)}{In(\frac{r_2}{r_1})}(r_1^2+\frac{4(r_2^2-r_1^2)}{In(\frac{r_2}{r_1})})(\frac{(-1+2In(r_1)).r_1^2+(1-2In(r_2)).r_2^2}{4.(r_1-r_2)}-\frac{In(r_1).(r_2^2-r_1^2)}{2}))+\frac{(r_2^2-r_1^2)}{K^2}\}]
\end{aligned}
\qquad (32)
$$

Substituting (32) in (29) yield:

$$\Delta P_\lambda=\int_0^1 (\frac{dp}{dz})dz \qquad (33)$$

$$F_\lambda^{(o)}=\int_0^1 r_2^2(-\frac{dp}{dz})dz \qquad (34)$$

$$F_\lambda^{(i)}=\int_0^1 r_1^2(-\frac{dp}{dz})dz \qquad (35)$$

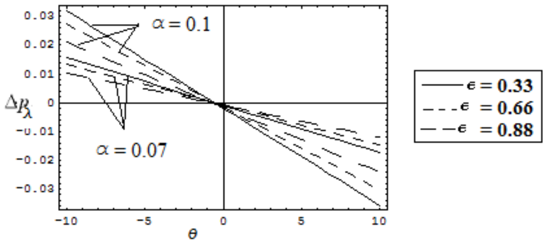
3. RESULT AND DISCUSSIONS

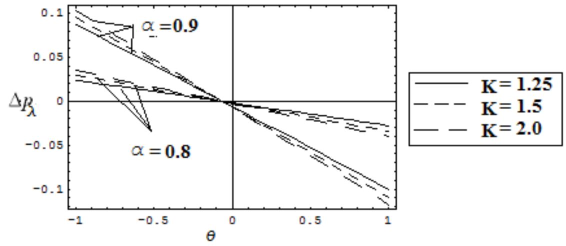
The dimensionless pressure rise (P_λ) and the friction forces on the inner and outer tube for different given values of the dimensionless flow rate Θ, amplitude ratio ϕ, radius ratio ε, γ is couple stress parameter, K porous media and viscosity parameter α are computed using the (33) to (35). As the integrals in (33) to (35) are not integrable in the closed form, so they are evaluated using

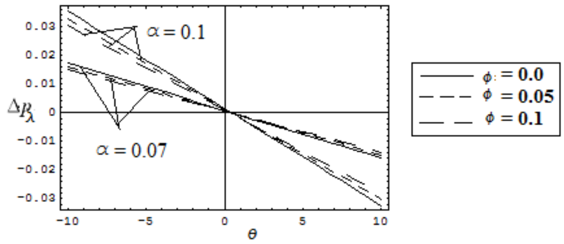
$$a_{20}=1.25cm, \qquad \frac{a}{\lambda}=0.156 \qquad (36)$$

The values of viscosity parameter α as reported by Srivastava et al. (1) are $\alpha = 0.0$ and $\alpha = 0.1$. Furthermore, since most routine upper gastrointestinal endoscopes are between 8–11mm in diameter as reported by Cotton and Williams (5) and radius ratio 1.25cm reported by Srivastava and Srivastava (7).

Fig. 1: shows the pressure rise against the flow rate; here it is observed that the pressure decreases with the increase of flow rate for different values of radius ratio ε = 0.33, ε = 0.66 *and* ε = 0.88 and pressure increases for the viscosity α = 0.07 and α = 0.1. **Fig2**: shows the pressure rise against the flow rate; here it is observed that the pressure increases with the increase of flow rate for different values of porous media K= 1.25, K= 1.5 *and* K= 2 and pressure increases for the viscosity α = 0.8 and α = 0.9. **Fig. 3**: shows that as the viscosity α increases the pressure increases. And for the different values of amplitude ratio ϕ = 0.0, ϕ = 0.05 and ϕ = 1, the pressure decreases. **Fig. 4**: it is noticed that the pressure decreases for different values of couple stress parameter γ = 0.4, γ = 0.6 and γ = 0.8. In **Fig. 5**: it is noticed that the friction force on the inner tube (endoscope) for different values of radius ratio ε = 0.32, ε = 0.38 *&* ε = 0.44 and for the values of viscosity α = 0.07 and α = 0.1. It is noticed that as the radius ratio ε increases the friction force on the inner tube increases and as the viscosity increases the friction force on the inner tube decreases. **Fig 6**: shows the friction force on the inner tube against the flow rate; here it is observed that the pressure increases with the increase of flow rate for different values of porous media K= 1.25, K= 1.5 and K = 2 and pressure increases for the viscosity α = 0.8 and α = 0.9. In **Fig.7**: it is noticed that the friction force on the inner tube (endoscope) for different values of amplitude ratio ϕ=0.0, ϕ=0.05& ϕ=1 and for the values of viscosity α = 0.07 and α = 0.1. It is noticed that the amplitude ratio ϕ increases the friction force on the inner tube decreases and as the viscosity increases the friction force on the inner tube increases. **Figures 8 and 12**: it is noticed that the viscosity decreases with friction force on the inner and outer tube increases in couple stress parameter γ. From **Figures 9, 10 and 11**: show the friction force on the outer tube for different values of radius ratio, amplitude ratio and Porous media; here it is observed that as radius ratio, amplitude ratio & porous medium increases the friction force increases.







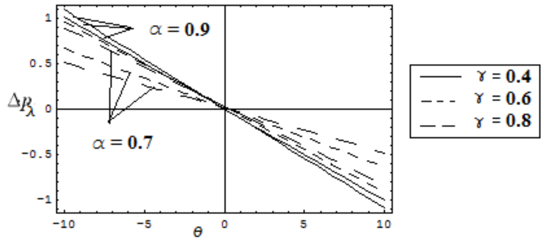
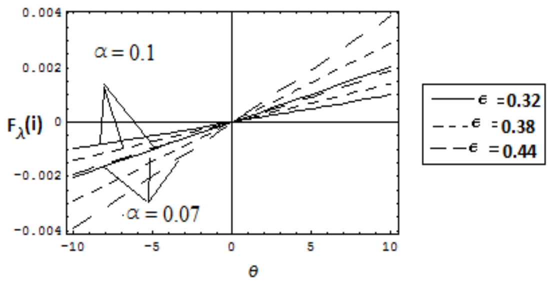
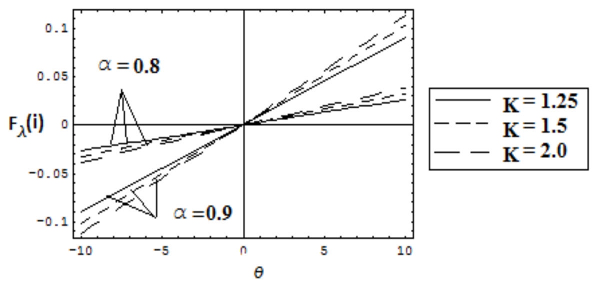
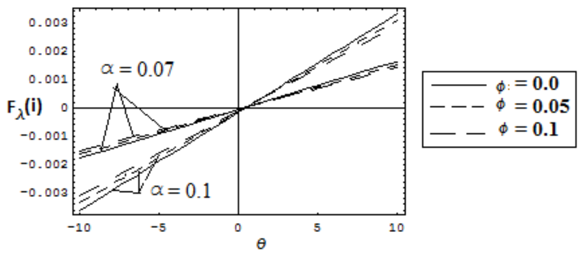


Fig1:Variation of Pressure rise over the flow rate for γ =0.4,K=2 ϕ=0,z=0.2 & different values of ε . Fig 2: Variation of Pressure rise over the flow rate for γ =0.4, ε=0.32,z=0.2, ϕ=0.4 & different values of K. Fig 3: Variation of Pressure rise over the flow rate for γ =0.4,K=5, ε=0.32,z=0.2 & different values of ϕ. Fig 4: Variation of Pressure rise over the flow rate for ε = 0.44, ϕ= 0.4, z= 0.2, K = 1.25 & different values of γ .







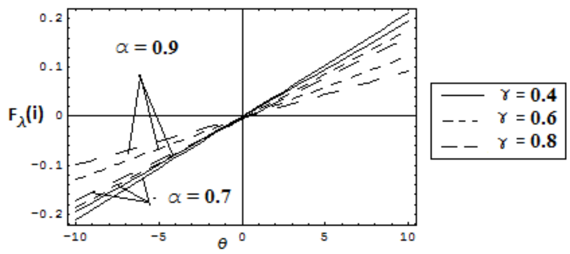


Fig 5: Variation of Friction on the inner tube (endoscope) over the flow rate for γ = 0.4, ϕ = 0.4,z =0.2, K=2 & different values of ε .Fig 6: Variation of Friction on

the inner tube (endoscope) over the flow rate for γ = 0.4, ε = 0.32, z = 0.2, ϕ= 0.4 & different values of K. Fig 7: Variation of Friction on the inner tube (endoscope) over the flow rate for γ=0.4, ε=0.32,z=0.2 ,K=2 & different values of ϕ. Fig 8: Variation of Friction on the inner tube (endoscope) over the flow rate for ε=0.44, ϕ=0.4, z=0.2,K=1.25 & different values of γ.

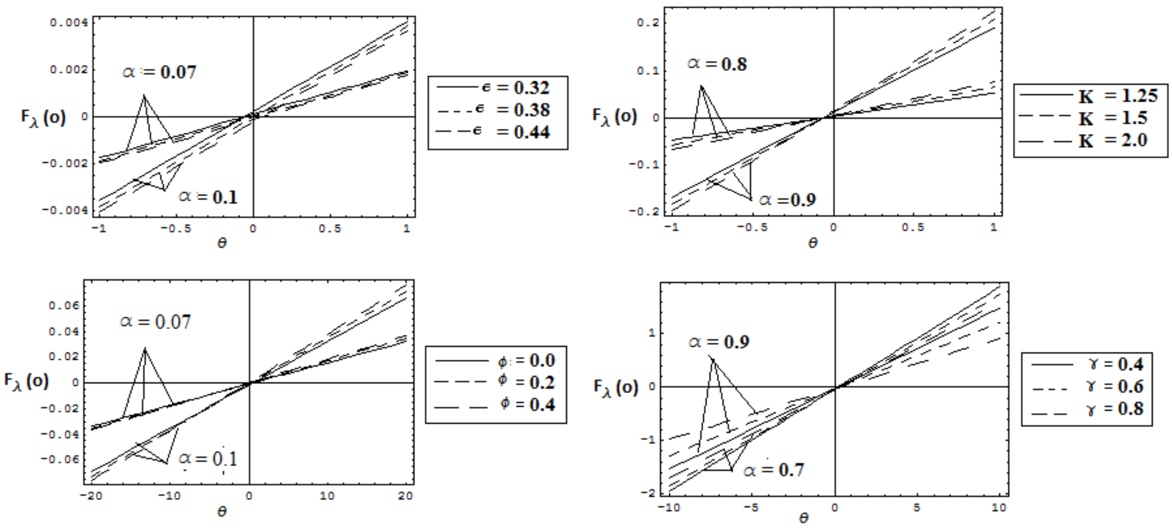


Fig 9: Variation of Friction on the outer tube over the flow rate for γ=0.4,ϕ=0.4,z=0.2,K=2 & different values of ε . Fig 10: Variation of Friction on the outer tube over the flow rate for γ=0.4, ε=0.32, z=0.2, ϕ=0.4 & different values of K. Fig11:Variation of Friction on the outer tube over the flow rate for γ=0.4, ε=0.32,z=0.2,K=2 & different values of ϕ. Fig 12: Variation of Friction on the outer tube over the flow rate for ε=0.44, ϕ =0.4,z=0.2,K=1.25 & different values of γ.

4. CONCLUSION

In this analysis peristaltic transport of a couple stress fluid in variable viscosity with porous medium been studied. The viscosity α increases with pressure rise increases for radius ratio ε, amplitude ratio ϕ, porous media K and couple stress parameter γ. The viscosity α increases with frictional forces of inner and outer tube increases for radius ratio ε, amplitude ratio ϕ, porous media K and couple stress parameter γ.

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